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application notes

Instantaneous Spectrum

Analysis with the Real-Time Third-Octave Analyzer

Edition 2

11 – 194

Instantaneous Spectrum Analysis with the Real-Time 1/3 Octave Analyzer Type 3347

Edition 2

Introduction

When a rapidly moving object is observed, its contours appear blurred to the eye. However, if the movement is periodic, the well known principle of stroboscopy can be employed to give an impression of a stationary condition. A stroboscopic flash light is synchronised with the periodic movement, and by varying the phase between some datum signal synchronised with the periodic movement and the actual signal which triggers the flash, the movement can be made to appear "frozen" in any required part of its cycle. Furthermore, if the trigger signal for the flash is put slightly out of synchronisation with the periodic frequency of the object, the eye gains an impression of a slow mechanical movement.

The same principle can be used to examine the changes occuring in electrical signals. The signals may be either periodic in nature like the sound from rotating machinery, or they may occur once only, such as the sounds used in speech. Single event signals can be made periodic by recording and then replaying them using a tape loop or a single event recorder such as the Type 7502. The frequency spectrum of the electrical signal can be investigated thoroughly by examining a short period of time during each cycle. This short period of examination during the long periodic change of the signal is analogous to the eye's observation while the light flashes during the periodic motion of the mechanical object. If frequency spectra from successive short periods during the cycle are displayed on a CRT screen, the same impression of slow motion will be obtained. This method of dividing the repeating signal into relatively short time periods for analysis is called the analysis of the Instantaneous Spectrum.

Division into short time periods can be accomplished by multiplying the original signal by some "window" function which has a value equal to zero except during the short period to be analysed. The shape of the window function is discussed in Appendix 1, which concludes that a gaussian shaped time weighting function gives several advantages over a simple square function. A special instrument, the Gauss Impulse Multiplier Type 5623, (Fig.1) has been constructed to produce the gaussian

time function and perform the multiplication. This instrument is described in a System Development Sheet.

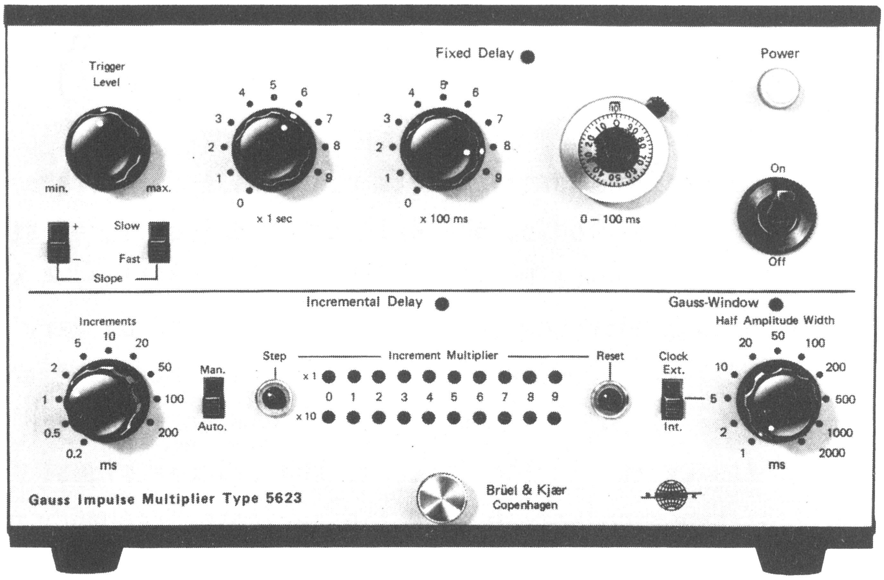


Fig. 1, The Gauss Impulse Multiplier Type 5623

Analysis of the Instantaneous Spectrum can yield valuable information about the way frequency content of a signal varies with time, and the method can be used to analyse any periodically repeating signal. This Application Note details how the Gauss Impulse Multiplier and Real-Time Third-Octave Analyser Type 3347 are used to perform the analysis. In this example the method was applied to speach analysis.

Speech Analysis

The measuring set-up used for Instantaneous Spectrum Analysis of speech is shown in Fig. 2.

A Tape Loop Adaptor was mounted on the Tape Recorder Type 7001, and a loop formed with length slightly longer than the original recorded signal. Selecting a tape speed of 60 inches per second to give a high repetition rate, the English word "THIS" was recorded on Channel 1 at a position shortly after the tape splice. A marker pulse was also recorded on the voice channel using the built-in marker gene-

rator. The start of the marker pulse was positioned on the tape a short distance before the voice record.

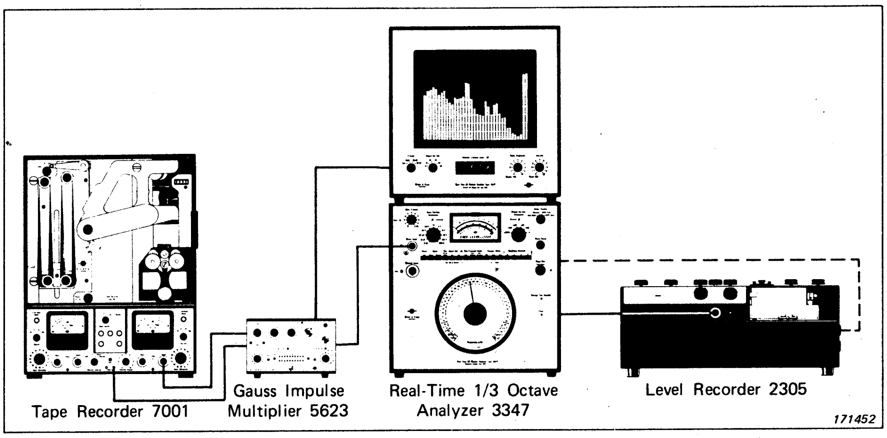
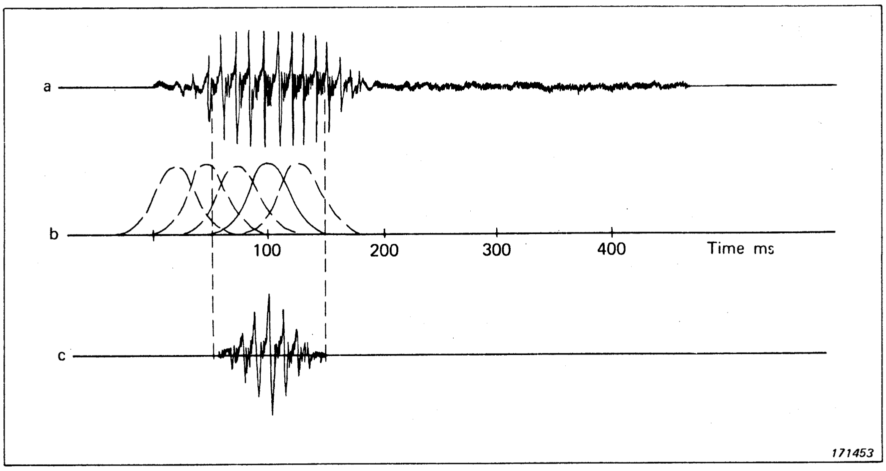


Fig. 2, The measuring set up used for analysis of the Instantaneous Spectrum

The recorded signal was then played back with a tape speed of 60 ips, and the marker pulse used to trigger the Gauss Impulse Multiplier. The signal fed to the Real-Time Analyzer from the Gauss Impulse Multiplier is the product of the recorded voice signal and the gaussian time function, and it appeared once for each cycle of the tape loop.

Fig. 3 shows the time varying signal of the spoken word, the gaussian shaped impulse and the result of multiplying the two together.



a. Amplitude-time function of the English word "this".

b. Gaussian weighted function used as time window.

c. Output from Gauss-multiplier.

Fig. 3, The gaussian signal.

The Real-Time Analyzer can be set either to average the readings when a reasonably high repetition rate can be obtained by using the "Slow Random" time constant (20 second signal averaging time) or to treat the incoming signal as a single impulse by using the "Sine" time constant and the "Store Max." function. This is described in Ref. 1. The addition of a small printed circuit card WB 0085 to the Real-Time Analyzer enables the "Store Max". function to be reset automatically by the trigger output signal from the Gauss Impulse Multiplier.

The width selected for the gaussian impulse will always be a compromise between the time and frequency resolution required as these are inversely related. In this case the half amplitude width chosen was 50 ms. The delay between the marker pulse and the actual triggering of the gaussian impulse was automatically increased 20 ms after each cycle of the tape loop so that the "slice" analysed was moved in 20 ms steps. Variation of the frequency content of the signal with time could easily be followed on the display screen with a new spectrum appearing after each revolution of the tape loop.

To obtain a hard copy of the results, the stored level in each position of the gaussian impulse was read out to the Level Recorder via the DC signal from the Multiplexer Output on the Real-Time Analyzer. In this case the delay on the Gauss Impulse Multiplier was stepped manually, and the automatic resetting of the Store Max function was interrupted so that the Recorder had sufficient time to complete the print-out. A total of 18 complete spectra were printed from the time period of the word; they are shown as a frequency/time/level "landscape" in Fig. 4.

The total length of the word is approximately 400 ms, and the spacing between spectra is 25 ms.

During the first part of the word, the "TH" sound gives a low frequency distribution with discrete frequencies in the 100 Hz third-octave band plus lower harmonics; this is easily seen in the photograph. 100 Hz is close to the "Chest Register" for men, which is the fundamental frequency of the vocal lips. The fundamental frequency of the spectra increases with time as the sound changes to "IS". These earlier parts of the word also contain frequencies that are more than one decade above the fundamental, but they appear to be related to the fundamental as they also increase with time.

In the later part of the word, the spectra are of a more random nature, with frequencies distributed around 4 kHz to 5 kHz. This is quite characteristic of the high

velocity airstream passing between teeth and tip of the tongue in the "SSS" sound.

This method of analysis is very useful in the study of how frequency content and amplitude vary with time, and can indicate the masking effects of discrete components in the spectra, described by E. Zwicker in Ref. 2.

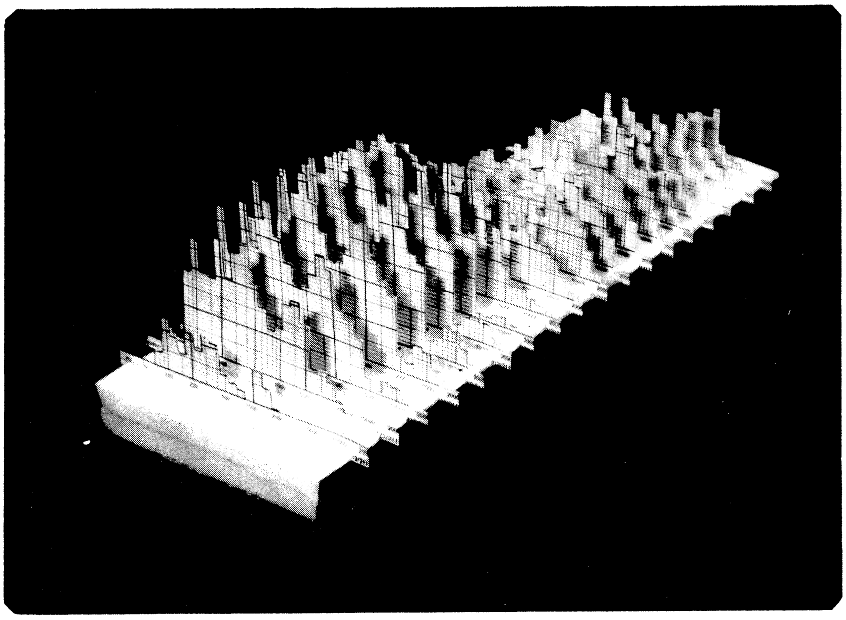


Fig. 4, Frequency/Time/Level Landscape.

Similar methods of analysis of the instantaneous spectrum can be applied to aircraft fly-over noise, sonic booms, explosions, analysis of heart sounds (Ref.3), traffic noise, measurement of vibration changes during one revolution of a machine, or vibration measurement during start-up and load or speed changes of a machine.

<u>REFERENCES:</u>

1. W.B.Frederiksen: 1/3 Octave spectrum readout of impulse measurements. Brüel & Kjær Technical Review No.1 1970.

2. E. Zwicker: "Die Verdeckung von Schmalbandgeräuschen durch Sinustöne. Acoustica 4, 1954, Beiheft 1.415.

3. Speech analysis. Spectrum strobing of sound. Analysis of heart sounds. Brüel & Kjær application notes No. 11-135.

Appendix 1: Instantaneous Spectrum Analysis

In the ordinary definition, the frequency spectrum of a given process is independent of time, since the spectrum gives the frequency content in the process as a whole, but it is often very informative to see how the frequency spectrum changes during the process. One way to get this information is to divide the process into short periods and analyse each one separately, analysis of the Instantaneous Spectrum.

Dividing a time varying function $f_1(t)$ into parts can be considered as multiplication with another time function $f_2(t)$ which is equal to zero except for a relatively short time. As a multiplication in the time domain results in a convolution of the frequency domain, expressed mathematically:

$$f_1(t) \cdot f_2(t) \leftarrow \rightarrow \int_{-\infty}^{\infty} F_1(\sigma) \cdot F_2(\omega - \sigma)\, d\sigma$$

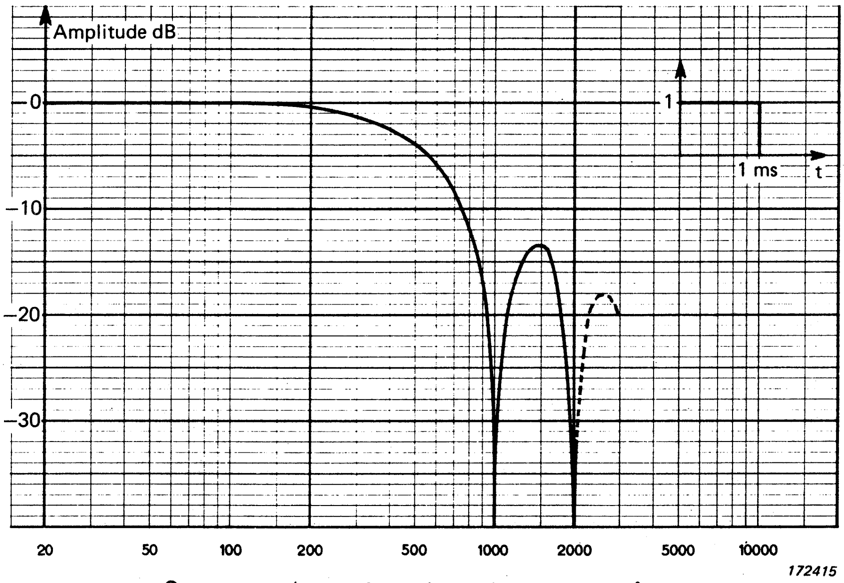
if: $f_1(t) \leftarrow \rightarrow F_1(\omega)$

$f_2(t) \leftarrow \rightarrow F_2(\omega)$

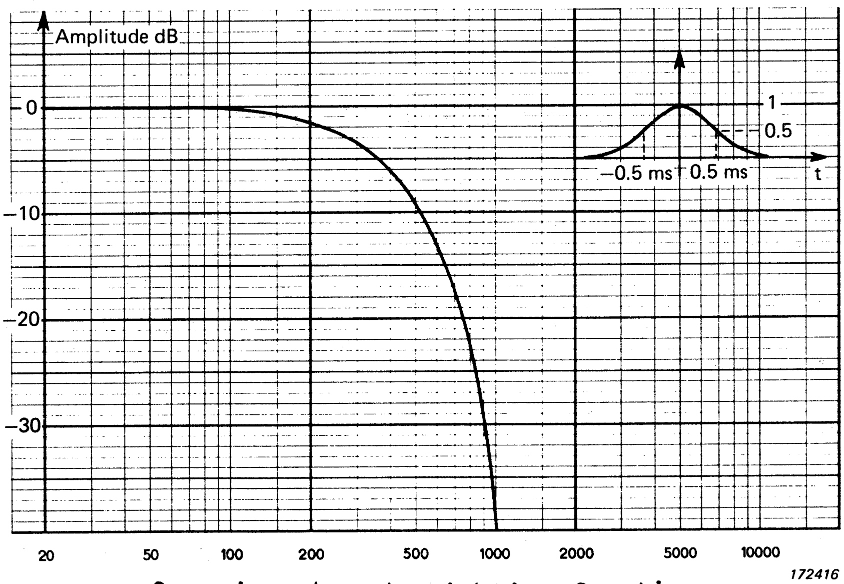
the instantaneous spectrum will be dependent on the frequency spectrum of the time weighting function used. In Fig. A1, two different weighting functions are shown together with their respective frequency spectra. The frequency spectra are plotted in Log-Log scale. A square shaped weighting function has the advantage of being easy to produce, because a multiplication simply means the signal is switched on when the square wave starts and switched off when it stops. However, the many side lobes in the frequency spectrum are a disadvantage. If a pure sinusoidal oscillation, represented by a single line in the frequency domain, is multiplied with a square shaped weighting function the instantaneous spectrum will not only show a maximum at the frequency of the line, but also at all the side maxima for both higher and lower frequencies. Another disadvantage is that the amplitude of these maxima depends on whether the weighting function starts and stops at a zero crossing of the sine wave signal.

In order to help overcome these disadvantages another weighting function, with gaussian shape has been considered.

$$f_2(t) = e^{-\alpha t^2}$$



Square shaped weighting function



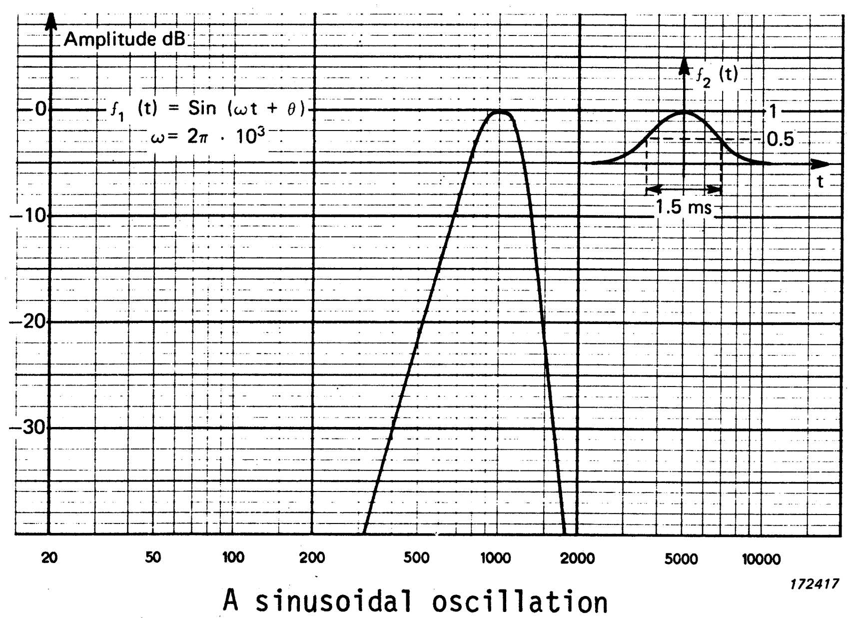
Gaussian shaped weighting function

Fig. A1,

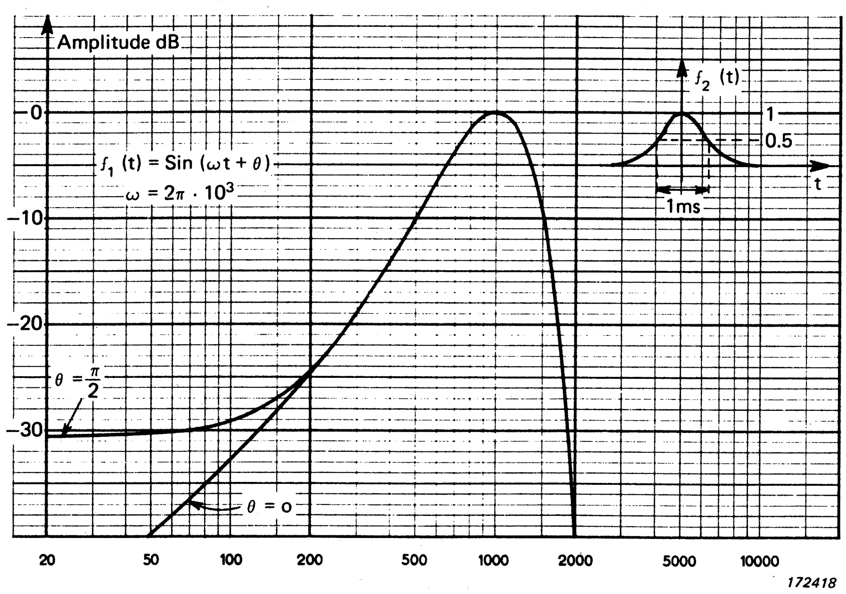
The advantage of using this function is that the frequency spectrum contains no side lobes at all, as the spectrum is also of gaussian shape.

$$F_2(\omega) = \frac{\sqrt{\pi}}{\sqrt{\alpha}} \cdot e^{-\frac{\omega^2}{4\alpha}}$$

In Fig. A2 a 1 kHz sinusoidal oscillation has been multiplied with a weighting funct of gaussian shape and the resulting spectrum calculated. It can be seen that the spectrum contains only one maximum, positioned at the frequency of the signal. Furth more, it can be seen that the instantaneous spectrum is almost independent of the time position of the weighting function relative to the sine wave, as long as the "half amplitude width" of the weighting function is equal to or larger than the period-time of the signal.



A sinusoidal oscillation



The result of multiplying the sine function with a gaussian shaped weighting function

Fig. A2,

Selecting the width of the weighting function is a compromise between the time and frequency resolution wanted, as they are inversely proportional.

To illustrate the use of instantaneous spectrum analysis a sine wave which starts at $t = 0$ has been investigated with a gaussian shaped weighting function. The signal $f_1(t)$ is defined as:

$$f_1(t) = 0 \qquad t < 0$$

$$f_1(t) = \sin(\omega_1 \cdot t) \qquad t > 0$$

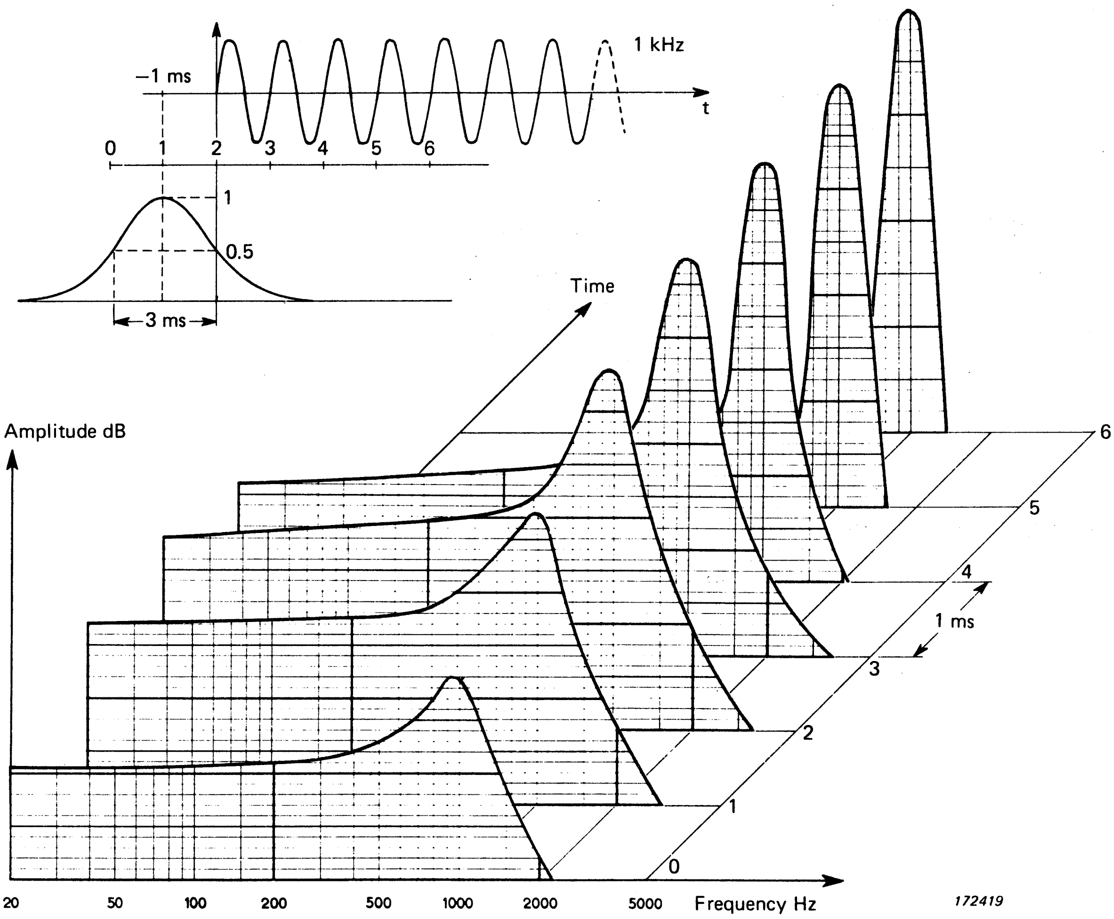
$$\omega = 2\pi \cdot 10^3$$

The weighting function $f_2(t)$ as:

$$f_2(t) = e^{-\alpha(t-t_0)^2}$$

$$\alpha = 0.3 \cdot 10^{-6} \qquad \text{(1/2 amplitude width = 3 ms)}$$

The calculations are made in accordance with the formulas given in Appendix 2, and the results are shown in Fig. A3.



The variation of the sine function with time when multiplied with a gaussian function

Fig. A3,

Mathematically speaking a sine wave has to last an infinitely long time to become a pure sinewave. In the case shown in Fig. A3, however, it is seen that after the duration of approximately 4 cycles of the sine wave, the instantaneous spectrum becomes constant, and the sine wave can be considered as pure.

If the weighting function is made wider the frequency resolution is improved and in this case the sinewave has to last more than 4 cycles to be considered as pure. In Appendix 2, all the formulas used for calculation are given, and it has been found that practical measurements fully support the theory.

APPENDIX 2: Formulas used for calculations

Fig. A1:

$$f(t) = 1 \quad ; \quad 0 < t < T$$

$$f(t) = 0 \quad ; \quad 0 > t > T$$

$$|F(\omega)| = T \frac{\sin \frac{\omega T}{2}}{\frac{\omega T}{2}} \quad ; \quad T = 1 \text{ ms.}$$

$$f(t) = e^{-\alpha t^2}$$

$$F(\omega) = \frac{\sqrt{\pi}}{\sqrt{\alpha}} e^{-\frac{\omega^2}{4\alpha}} \quad ; \quad \alpha = 2.75 \cdot 10^6$$

Fig. A2:

$$f_1(t) = \sin(\omega_1 t + \theta) \quad ; \quad \omega_1 = 2\pi \cdot 10^3$$

$$f_2(t) = e^{-\alpha t^2} \quad ; \quad \alpha = \left(\begin{array}{l} 2.75 \cdot 10^6 \\ 1.23 \cdot 10^6 \end{array} \right.$$

$$|F(\omega)| = \sqrt{R^2(\omega) + X^2(\omega)}$$

$$R(\omega) = \sin(\theta) \frac{\sqrt{\pi}}{2\sqrt{\alpha}} \left(e^{-\frac{(\omega_1 + \omega)^2}{4\alpha}} + e^{-\frac{(\omega_1 - \omega)^2}{4\alpha}} \right)$$

$$X(\omega) = \cos(\theta) \frac{\sqrt{\pi}}{2\sqrt{\alpha}} \left(e^{-\frac{(\omega_1 - \omega)^2}{4\alpha}} - e^{-\frac{(\omega_1 + \omega)^2}{4\alpha}} \right)$$

Fig. A3: For the calculations in Fig. A3, the sine wave $f_1(t)$ has been considered as a toneburst with 8 cycles.

$$f_1(t) = 0 \quad ; \quad -4\,\text{ms} > t > 4\,\text{ms}$$

$$f_1(t) = \sin(\omega_o t) \quad ; \quad -4\,\text{ms} < t < 4\,\text{ms}$$

$$F_1(\omega) = \frac{2 \cdot \sin\left(8\pi \frac{\omega}{\omega_o}\right)}{j\,\omega_o\left(\left(\frac{\omega}{\omega_o}\right)^2 - 1\right)} \quad ; \quad \omega_o = 2\pi \cdot 10^3$$

$$f_2(t) = e^{-\alpha(t-t_o)^2} \quad ; \quad \alpha = 0.3 \cdot 10^6$$

$$F_2(\omega) = \frac{\sqrt{\pi}}{\sqrt{\alpha}} \cdot e^{-\frac{\omega^2}{4\alpha}} \cdot e^{-j\omega t_o}$$

$$f_1(t) \cdot f_2(t) \leftarrow \rightarrow F(\omega)$$

$$F(\omega) = \int_{-\infty}^{\infty} \left(\frac{2 \cdot \sin\left(8\pi \frac{\sigma}{\omega_o}\right)}{j\,\omega_o\left(\left(\frac{\sigma}{\omega_o}\right)^2 - 1\right)} \cdot \frac{\sqrt{\pi}}{\sqrt{\alpha}} \cdot e^{-\frac{(\omega-\sigma)^2}{4\alpha}} \cdot e^{-j(\omega-\sigma)t_o} \right) d\sigma$$

The first spectrum shown in Fig. A3 is calculated for $t_o = -6\,\text{ms}$.

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